Chapter 8

Similarity

Section 2
Problem Solving in Geometry with Proportions

GOAL 1: Using Properties of Proportions

In Lesson 8.1, we studied the reciprocal property and the cross product property. Two more properties of proportions, which are especially useful in geometry, are given below.

ADDITIONAL PROPERTIES OF PROPORTIONS

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Example 1: Using Properties of Proportions

Tell whether the statement is true.

a. If $\frac{p}{6} = \frac{r}{10}$, then $\frac{p}{r} = \frac{3}{5}$.

$\frac{p}{r} = \frac{6}{10} \rightarrow \frac{3}{5}$

yes

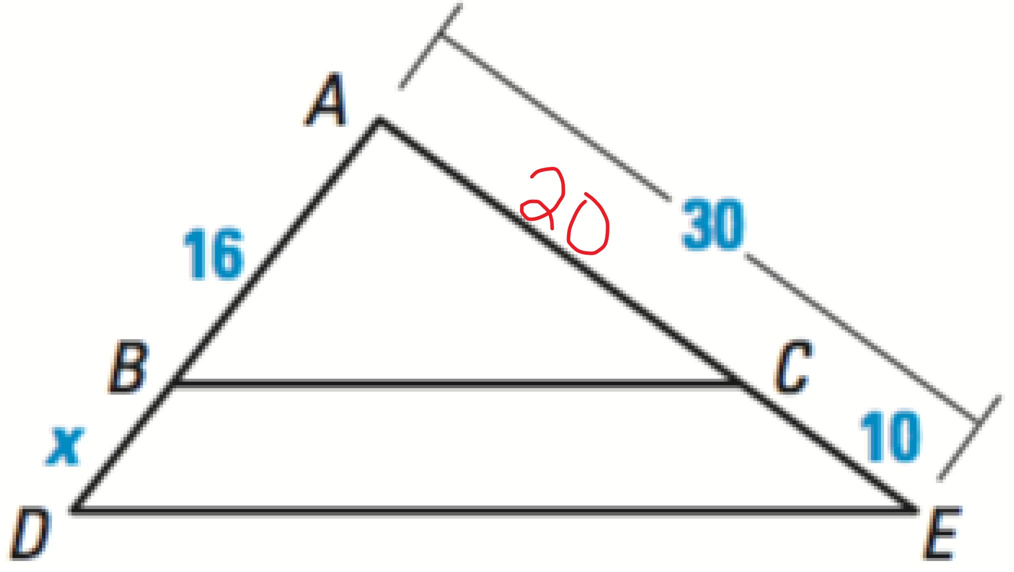
b. If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+\cancel{3}}{4}$.

no

$\frac{a+3}{3} = \frac{c+4}{4}$

Example 2: Using Properties of Proportions

In the diagram $\frac{AB}{BD} = \frac{AC}{CE}$. Find the length of $\overline{BD}$.



$$\frac{16}{x} = \frac{20}{10}$$

$$\frac{\cancel{20}x}{\cancel{20}} = \frac{160}{20}$$

$$x = 8$$

The **geometric mean** of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. If you solve this proportion for x, you find that $x = \sqrt{a \cdot b}$, which is a positive number.

For example, the geometric mean of 8 and 18 is **12**, because $\frac{8}{12} = \frac{12}{18}$, and also because $\sqrt{8 \cdot 18} = \sqrt{144} = 12$.

Example 3: Using a Geometric Mean

Paper Sizes: International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A4 and A3. The distance labeled x is the geometric mean of 210 mm and 420 mm. Find the value of x.

$$x = \sqrt{a \cdot b}$$

$$x = \sqrt{210 \cdot 420}$$

$$x = \sqrt{88200}$$

$$x = 296.98 \text{ mm}$$

$$\frac{210}{x} = \frac{x}{420}$$

$$\sqrt{x^2} = \sqrt{88200}$$

$$x = 296.98 \text{ mm}$$

GOAL 2: Using Proportions in Real Life

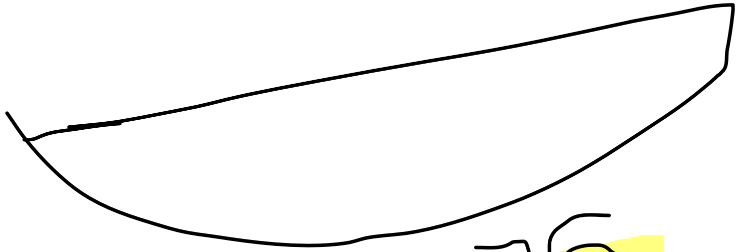
In general, when solving word problems that involve proportions, there is more than one correct way to set up the proportion.

Example 4: Solving a Proportion

MODEL BUILDING A scale model of the Titanic is 107.5 inches long and 11.25 inches wide. The Titanic itself was 882.75 feet long. How wide was it?

scale

actual?



882.75

$$\frac{107.5}{882.75} = \frac{11.25}{x}$$

$$\frac{\cancel{107.5}x}{\cancel{107.5}} = \frac{9930.9375}{107.5}$$

$$x = 92.38 \text{ ft}$$

Notice that the proportion in Example 4 contains measurements that are not in the same units. When writing a proportion with unlike units, the numerators should have the same units and the denominators should have the same units.

EXIT SLIP